INSTITUT FÜR ANGEWANDTE ANALYSIS UND NUMERISCHE SIMULATION

mathematik ians

Universität Stuttgart

Vortragsankündigung Oberseminar

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10:00 Uhr im Raum 8.122

30.05.2018 **Andreas Stein, M.Sc.** (Universität Stuttgart)

Uncertainty Quantification beyond the Gaussian case

Abstract: Uncertainty quantification plays an increasingly important role in a wide range of problems in the Engineering Sciences and Physics. Examples of sources of uncertainty are imprecise or insufficient measurements and noisy data. In the underlying dynamical system this is in general reflected via a stochastic operator and/or stochastic data. These parameters are often modeled as time-space Gaussian processes, leading to continuous random functions and thin-tailed, symmetric Gaussian distributions. For several applications, however, it might be favorable to model the stochastic quantities as discontinuous processes which also allow for asymmetric and heavy-tailed distributions.

For instance, as a simplified model for subsurface flows second order elliptic equations with random coefficients may be utilized. Insufficient measurements or uncertainty in those are modeled by a random coefficient, which then accounts for the uncertain permeability of a given medium. To represent transitions in heterogeneous\fractured\porous media, we model the coefficient as the sum of a (continuous) Gaussian random field and a (discontinuous) jump part. Moments of the solution to the resulting random partial differential equation are then estimated by a pathwise numerical approximation combined with multilevel Monte Carlo (MLMC) sampling. In order to account for the discontinuities and improve the convergence of the pathwise approximation, the spatial domain is decomposed with respect to the jump positions in each sample, leading to pathdependent grids. Hence, it is not possible to create a nested sequence of grids which is suitable for each sample path a-priori. We address this issue by an adaptive multilevel algorithm, where the discretization on each level is sample-dependent and fulfills given refinement conditions. As we will show, the adaptive MLMC algorithm may be readily applied to a time-dependent parabolic setting with random discontinuous advection and diffusion coefficients.

As a second example, we consider stochastic partial differential equations with driving noise term given by the source function on the right hand side. The noise is given by an infinite-dimensional Lévy process (or Lévy field), i.e. a Hilbert-space valued stochastic process with temporal discontinuities. Such equations arise for example in the valuation of energy forward contracts, where the spot price is modeled by a stochastic transport problem. We focus on the numerical approximation of the Lévy field and employ Karhunen-Loève expansions to obtain a spectral representation with respect to the covariance operator of the field. For square integrable fields beyond the Gaussian case, the one-dimensional distributions in this expansion are no longer independent, but merely uncorrelated. The dependence structure among the one-dimensional processes ensures that the resulting field exhibits the correct point-wise marginal distributions. We derive this point-wise distributions for a specific class of subordinated Wiener processes in closed form. Further, to approximate the respective (one-dimensional) Lévy-measures in the spectral expansion, a numerical method, called discrete Fourier inversion, is developed. For this method, L^p-convergence rates can be obtained and, under certain regularity assumptions, mean square and L^p-convergence of the approximated field is proved. Numerical examples, which include hyperbolic and normal-inverse Gaussian fields, demonstrate the efficiency of the approach.

This is joint work with Andrea Barth (SimTech, University of Stuttgart)

having a coffee in a Banach space makes you feel complete

```
typedef Dune::ACFem::MassModel<EllipticModelType> MassModelType;
MassModelType bareMassModel(implicitEllipticModel);

auto massModel(mu * (mat.Z_a) * J + mat.Z_w) * bareMassModel);
```

Alle Interessenten sind herzlich eingeladen!

Die Professoren des Instituts für Angewandte Analysis und Numerische Simulation

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