Fault-Tolerant Multigrid Algorithms Based on Error Confinement and Full Approximation

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The resilience challenge

Some important observations

- More components at exascale $\Rightarrow$ higher probability of failure
- Active debates to sacrifice reliability for energy efficiency
- MTBF $< 1\,\mathrm{h}$ $\Rightarrow$ all simulation software must be prepared

Classical techniques

- Reliability in hardware (ECC protection etc.) too power-hungry
- Checkpoint-restart too memory-intensive (and too slow)
- Triple modular redundancy too power-hungry, but: can be more energy-efficient and faster for large fault rates

Focus of this talk: ABFT for multigrid

General concept: algorithm-based fault tolerance

- Exploit algorithmic properties to detect and correct faults
- Can be more efficient than middleware

Previous work

- Scenario: Node loss
- Self-stabilisation properties

In this talk: Silent Data Corruption

- Scenario: Bitflips
- Black-box smoother protection

Resilient multigrid

Silent data corruption

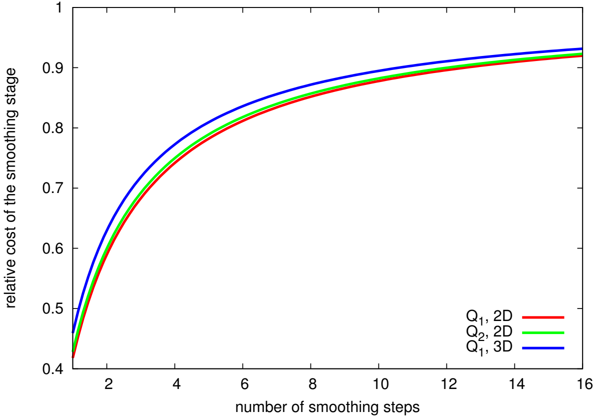
Silent data corruption

- Soft transient faults that lead to wrong solutions
- Sometimes noticeable a posteriori (divergence), mostly not
- Causes: radiation, leaking voltage, silicon ageing, ...

Core idea of our approach

- Use FAS multigrid to increase robustness
- Based on nonlinear MG: not just a correction on each level but a true approximation of the solution
- Linear case: numerically equivalent, less than one fine SPMV overhead per cycle
- One additional vector necessary

Focus on smoother stage



- Relative cost of the smoother stage as a function of the number of smoothing steps
- Detailed model: in the paper

Black-box smoother protection

Theoretical justification for the down-cycle

- Obvious: residual $\mathbf{r}$ converges to zero on finest grid
- Easy to prove: residual (monotonously) converges to zero on all grids

Theoretical justification for the up-cycle

- Slightly non-trivial proof: correction vector $\mathbf{c}$ converges (monotonously) to zero on all grids

Consequence: good fault indicators

- Both readily available without additional computation

Black-box smoother protection

Practical realisation after smoothing on level k

- Compute index set of possibly faulty components of $\mathbf{c}$ or $\mathbf{r}$ by comparing against level-specific threshold from earlier iteration
- Extend by one layer of indices coupled by $\mathbf{A}$
- Replace faulty components by unsmoothed values (down-cycle), or by recomputed correction from (non-faulty) coarser grid
- Adaptively update threshold

Black-box smoother protection

$\mathcal{L} = \texttt{detect\_and\_localise}(c, t_k^c)$ // detect faulty components

if $\mathcal{L} \neq \emptyset$ **then**

  $\tilde{v} = \mathbf{I}_k^{k-1} v^{(0)}$ // restrict (non-faulty) initial approximation

  $\tilde{c} = u_{k-1} - \tilde{v}$ // calculate new correction vector

  $\tilde{c} = \mathbf{P}_{k-1}^k \tilde{c}$ // prolongate new correction vector

  for $i \in \mathcal{L}$ **do**

    $c(i) = \tilde{c}(i)$ // replace faulty components

  end

  $t = \texttt{calc\_threshold}(c)$ // calculate new threshold value

  $q = t / t_k^c$ // relative reduction of threshold value

  if $q > 1$ // rescale other threshold values

  then

    for $i \in \{0, \dots, L\} \setminus \{k\}$ **do**

      $t_i^c = t_i^c \cdot 10 \cdot q$

    end

  end

  $t_k^c = t$ // store threshold value

else

  $t_k^c = \texttt{calc\_threshold}(c)$ // calculate and store threshold value

end

Checksum protection for transfer stage

Checksums

- Use identity $\mathbf{1}^\mathsf{T}(\mathbf{A}\mathbf{x}+\mathbf{y}) = (\mathbf{1}^\mathsf{T}\mathbf{A})\mathbf{x} + \mathbf{1}^\mathsf{T}\mathbf{y}$
- Precompute $\mathbf{1}^\mathsf{T}\mathbf{A}$ (column sums)
- Fault detection in $\mathbf{A}\mathbf{x}+\mathbf{y}$ by three dot products
- More elaborate schemes: detect and correct errors

Combined approach

- Black-box smoother protection, checksums for the rest
- Runtime comparison, fault-free case

	unprotected	**transfer stage** (checksums)	**smoothing stage** (new algorithm)	**FTMG** (both)
mean time	26.1562	27.4128	26.7905	27.9090
factor	1	1.0480	1.0243	1.0670

Numerical experiments

Test problems

- Different flavours of anisotropic diffusion-convection-reaction
- 2D, Q_1 FEM, eight levels, 263169 DOF

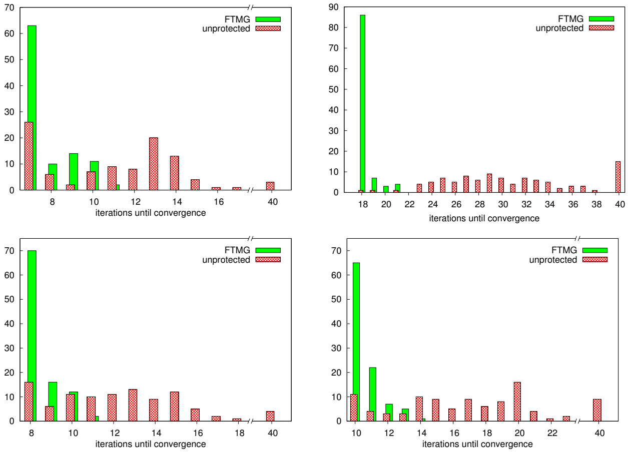
Solver configuration

- FAS-MG, V-cycle, 4+4 Jacobi $\omega = 0.7$ smoothing steps
- Bilinear interpolation for restriction and prolongation
- Natural injection for approximations

Fault injection

- 1 % bitflip probability in current approximation after each smoothing step on each level (deliberately unrealistically high)
- Actual component and bit fully random
- 100 repetitions per experiment

Numerical experiments



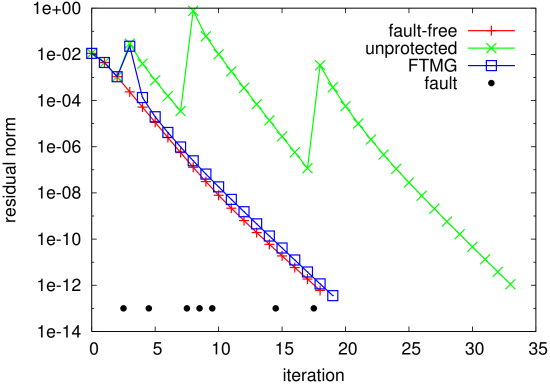
Histograms of iterations for poisson, anisotropic diffusion (andi), diffusion-convection (dico) and anisotropic diffusion-convection-reaction (andicore)

Numerical experiments (Summary)

		#detects		#iterations			
problem	**#faults**	**direct**	**delayed**	**fault-free**	**FTMG**	**unprotected**	
`poisson`	3.82	1.89	0.61	7	7.79	10.82	(3)
`andi`	10.24	4.94	0.49	18	18.25	29.09	(15)
`dico`	4.92	2.01	0.39	8	8.46	11.89	(4)
`andicore`	5.77	2.40	0.25	10	10.55	16.14	(9)

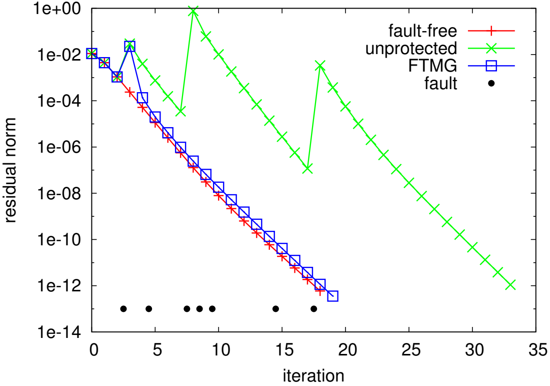
- Divergent runs in brackets
- Some faults not directly detected: delayed repair with ‘faultier’ correction

Detailed analysis of one andi test



- Blue bump: delayed repair with already faulty correction
- Two other significant faults: well repaired

Detailed analysis of one andi test



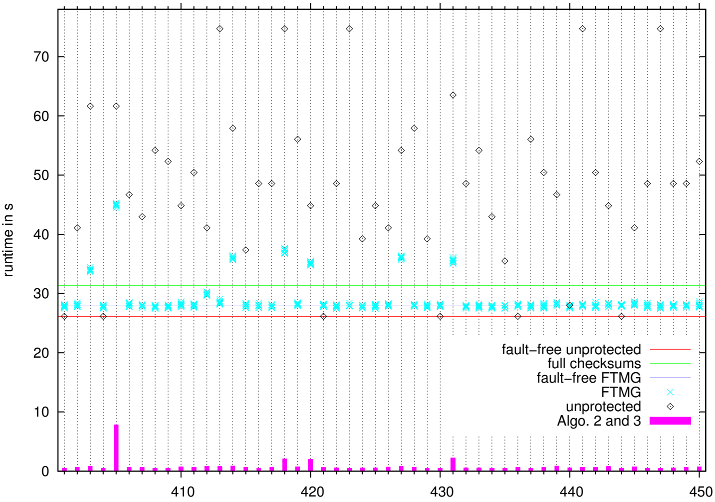
iter	cycle	level	step	change (%)	#det	#corr
3	up	6	1	9.99e+1	9	31
8	up	7	3	9.38e+1	25	49
18	down	7	4	1e+2	9	25

Detailed analysis of one andi test

iter	cycle	level	step	change (%)	#det	#corr
3	down	5	3	2.14e-4		
3	down	5	4	2.72e-11		
3	up	6	1	9.99e+1	9	31
5	up	5	3	5.55e-10		
8	down	7	3	1.81e-9		
8	up	7	3	9.38e+1	25	49
9	down	4	2	6.75e-3	49	81
9	up	1	2	4.47e-4	17	45
10	up	5	2	0		
15	up	1	1	1.25e-4	28	54
18	down	7	4	1e+2	9	25

- Irrelevant faults (small changes) are tolerated
- Small additional corrections (red)

Performance results



Summary

Summary and acknowledgements

- Fault tolerance becomes increasingly important
- Black-box ‘computer science’ techniques exist and work well, but: substantial overhead
- ABFT techniques may do better
- In this talk: some (early) ideas for multigrid

- Papers and more information
  - Silent data corruption: almost submitted
  - Minimised checkpointing: Parallel Comp. 49:117–135, 2015

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