University of Stuttgart
Department of Mathematics

Fault-tolerance for linear solvers with a focus on multigrid

ESCAPE 2 – Workshop on fault tolerant algorithms and resilient approaches for exascale computing

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SimTech

Focus of our research

- Faults introduced by *silent data corruption* (SDC): Stored data is not changed but the result of a computation, e.g., for $a = b = 2$ we receive $c = a + b = 5$
- Node-losses which are a variation of hard faults/failures

Classical techniques

- Reliability in hardware (ECC protection etc.) too power-hungry
- Checkpoint-restart too memory-intensive (and too slow)
- Triple modular redundancy too power-hungry, but: can be more energy-efficient and applicable for large fault rates

Another approach

Algorithm-based fault-tolerance

Algorithm-based fault-tolerance

- Exploit algorithmic properties to detect and correct faults on-the-fly
- Can be more efficient than middleware-based solutions

  Benefit: Provable error bounds possible

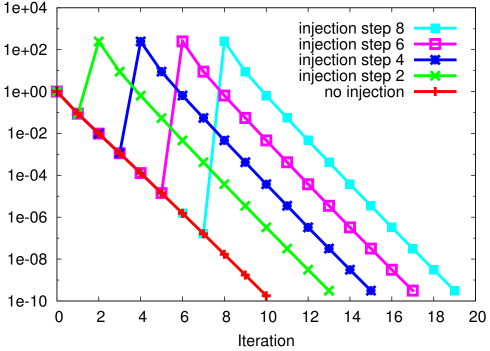
Challenges

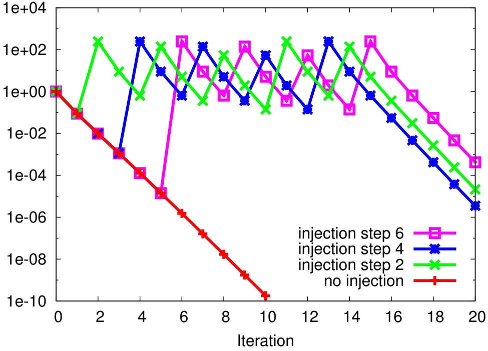
- Requires custom modifications for each class of methods
- Overhead in the fault-free scenario should be small
- False-positives should be rare without much impact on convergence
- MPI: Faults can result in node-losses
  $\Rightarrow$ Once a rank dies, the universe is dead in current MPI3

Initial focus

Multigrid because it is an optimal solver for elliptic problems.

Multigrid and faults

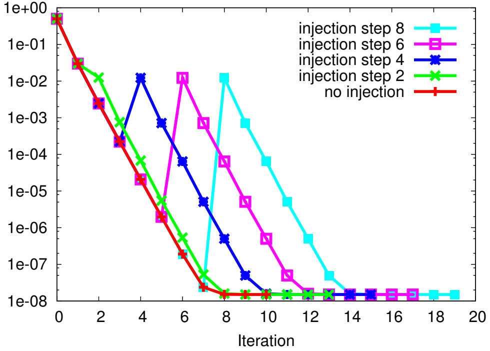


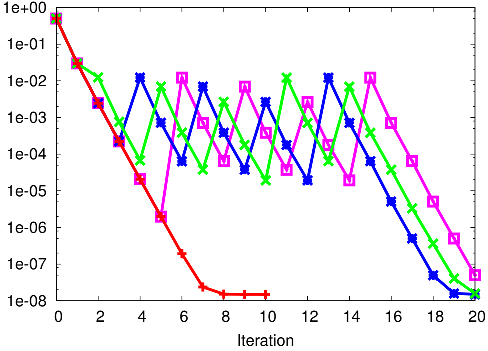


Observations (node-loss)

- A fault is comparable to a restart of the multigrid solver
- Multigrid converges always if the fault-rate is not to high
- Note: SDC and node-losses results in similar behaviour

Multigrid and faults





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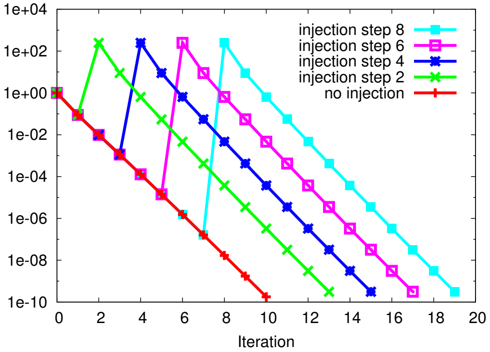

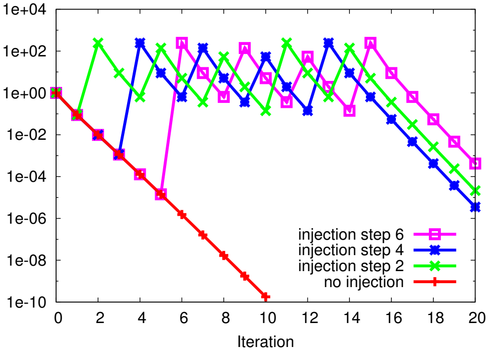

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- Multigrid converges always if the fault-rate is not to high
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⇒ Multigrid is provable self-stabilising:
Starting from any state the algorithm comes back to a valid state and eventually converges afterwards

Ongoing projects

❶ Compressed checkpointing

- Using compression techniques to decrease checkpoint size
- Locally restore lost or faulty data from compressed checkpoint
- Improve restoration by solving local auxiliary problems

❷ SDC-tolerant multigrid

- Increase the inherent robustness of multigrid with respect to bit-flips
- Apply a local smoothing stage protection to detect and repair soft faults

❸ User level exception handling

- Propagate exceptions with MPI to always ensure same state on all ranks
- Necessary for an efficient implementation of the other projects
- User-friendly asynchronous C++ MPI interface for parallel exception handling

Compressed checkpointing

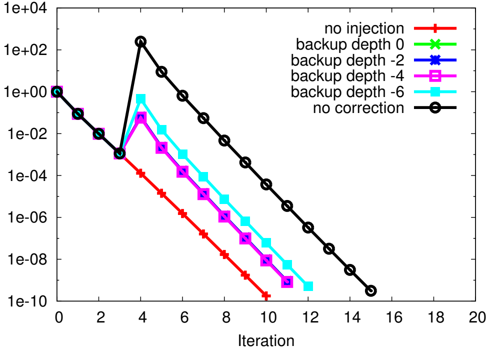
Research goal

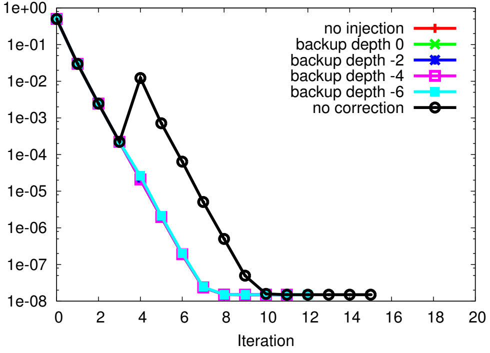
Reduce size of checkpoints and restore lost data efficiently

❶ Compressed checkpointing

Multigrid compression

- Use multigrid transfer operators to compress checkpoint
- Data reduction in d dimensions: $\sim 2^d$ per level (backup depth)
- Restore lost data with prolongated checkpoint





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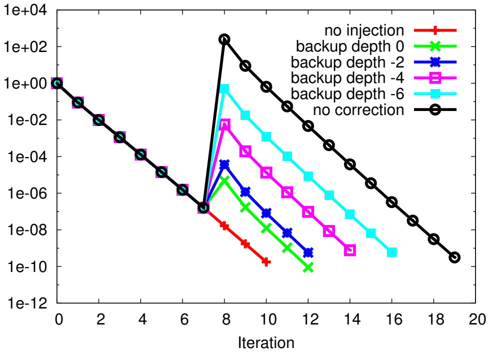

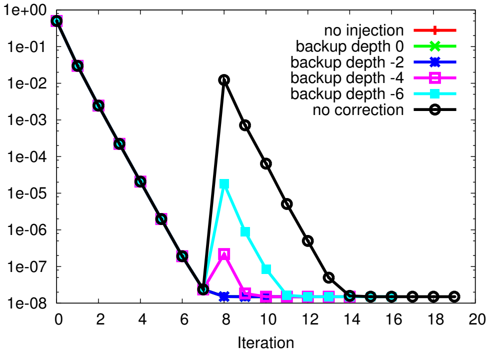

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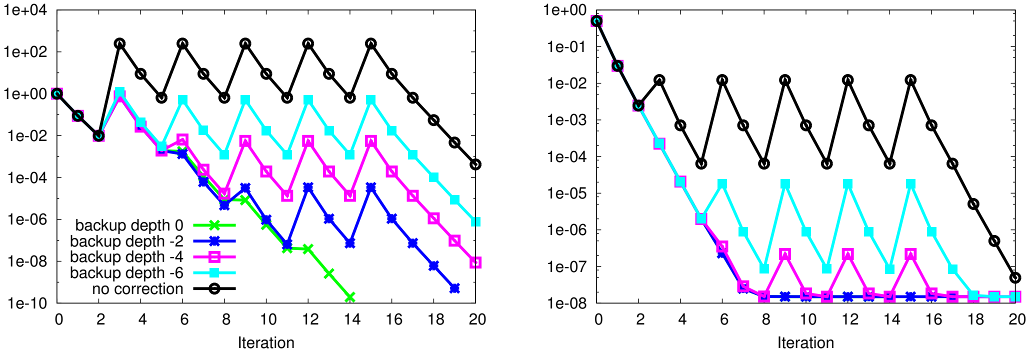
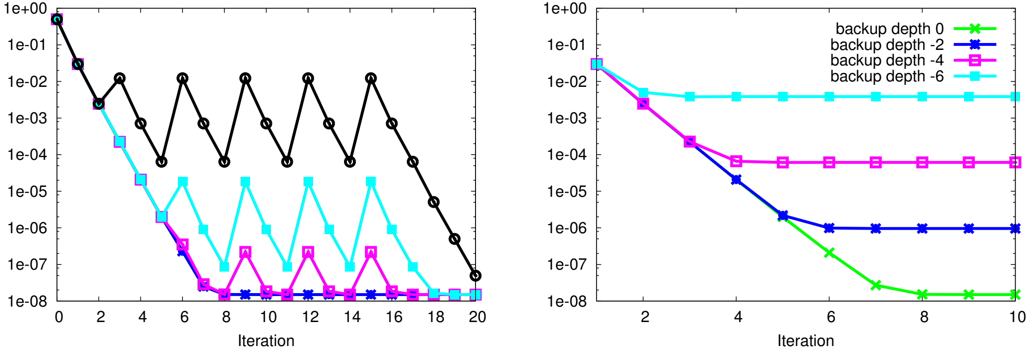


❶ Compressed checkpointing

Limits of multigrid compression



- Discretisation error dominates at some point
- Dominates earlier for highly compressed data
- Factor between L^2-quality and L^2-error depends on amount of repaired data

❶ Compressed checkpointing

Problem

- Late faults: Convergence cannot be restored with highly compressed backups
- Recurrent faults need even less-compressed checkpoints

Solution

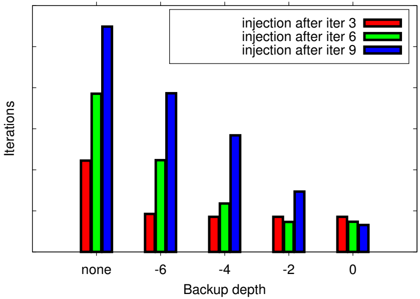
- Solve an auxiliary problem with Dirichlet boundary to improve accuracy
- Use decompressed data as initial guess

Auxiliary problem (compare Huber et al.[1])

Extend faulty indices $\mathcal{F} \subset \mathbb{N}$ by connectivity pattern of Operator $\mathbf{A}$ to $\mathcal{J}$ and solve

$$\begin{aligned} \mathbf{A}(\mathcal{J},\mathcal{J})\tilde{x}(\mathcal{J}) &= b(\mathcal{J}) && \text{in } \mathcal{F} \\ \tilde{x} &= x && \text{on } \mathcal{J}\setminus\mathcal{F} \end{aligned}$$

iteratively with initial guess $\tilde{x} = x_{cp}$ in $\mathcal{F}$.



[1] M. Huber, B. Gmeiner, U. Rüde, B. Wohlmuth, **Resilience for Massively Parallel Multigrid Solvers**, SIAM Journal on Scientific Computing, 2016

❶ Compressed checkpointing

Summary: Multigrid compression

- Multigrid compressed checkpoints can be used to recover from faults
- Early fault: Highly compressed data is sufficient
- Late fault: Compression rate has to be decreased
- Eventually an auxiliary problem has to be solved to ensure convergence
- The decompressed data is a good initial guess for this auxiliary problem
- Same idea could be used with other hierarchic methods

But

Multigrid is good preconditioner, but rarely a standalone solver

D. Göddeke, M.A., D. Ribbrock, **Fault-tolerant finite-element multigrid algorithms with hierarchically compressed asynchronous checkpointing**, Parallel Computing, 2015

❶ Compressed checkpointing

Aim: Extend the idea to other solvers and methods

Restore lost/faulty data from different kind of checkpoints

- Checkpoints based on different (lossy) compression techniques:
  - Multigrid compression
  - SZ compression
- Checkpoints with different frequency
- Different restoration approaches
  - Local restoration based on compressed checkpoint
  - Global roll-back to compressed checkpoint
  - Improved restoration by solving auxiliary problem

Task

Compare quality and efficiency of repair using different combinations

❶ Compressed checkpointing

SZ compression

- Save initial point value (with reduced accuracy): unpredictable data
- Predict next point based on previous processed points via an interpolation based on n layers
- Compare predicted value with real value and improve it through a *Huffman*-like coding
- If still not ‘close enough’ data is stored as unpredictable and compressed via binary-representation analysis

Advantages and disadvantages

- Adaptive controllable compression rate (via parameter)
- More computational overhead than inherent multigrid compression
- No random-access to single decompressed values

D. Tao, S. Di, Z. Chen and F. Cappello, **Significantly Improving Lossy Compression for Scientific Data Sets Based on Multidimensional Prediction and Error-Controlled Quantization**, Computing Research Repository, 2017

❶ Compressed checkpointing

Numerical tests

- Anisotropic diffusion in 2D

$$-\nabla \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0.01 \end{pmatrix} \nabla u = b$$

- Linear finite elements on grid with 1500×1500 degrees of freedom
- Solver: 4-way parallel CG without overlap (slices)
- Preconditioner: Algebraic multigrid (one V-cycle)
- MG compression of 3 levels; SZ compression tolerance is adaptive:

$$\texttt{locale\_def\_norm\_at\_backup\_time} \;/\; \texttt{def.size()} * 10^{-3}$$

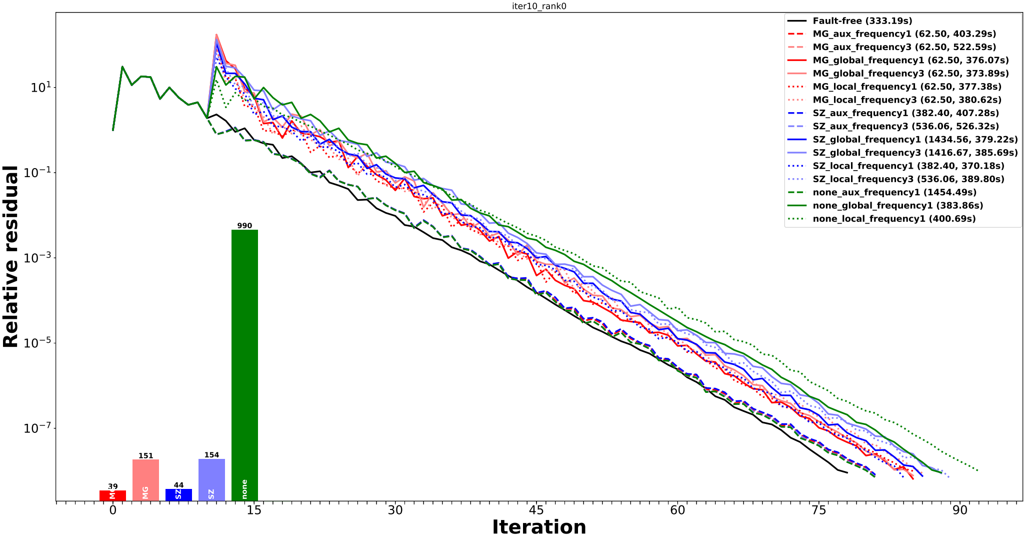
- Auxiliary solver reduction to

$$\texttt{global\_def\_norm\_at\_backup\_time} \;/\; \sqrt{(\texttt{\#cores})} * 10^{-(\texttt{age\_of\_backup}+1)}$$

Challenge

Recover from a fault in the solver with a compressed checkpoint

❶ Compressed checkpointing

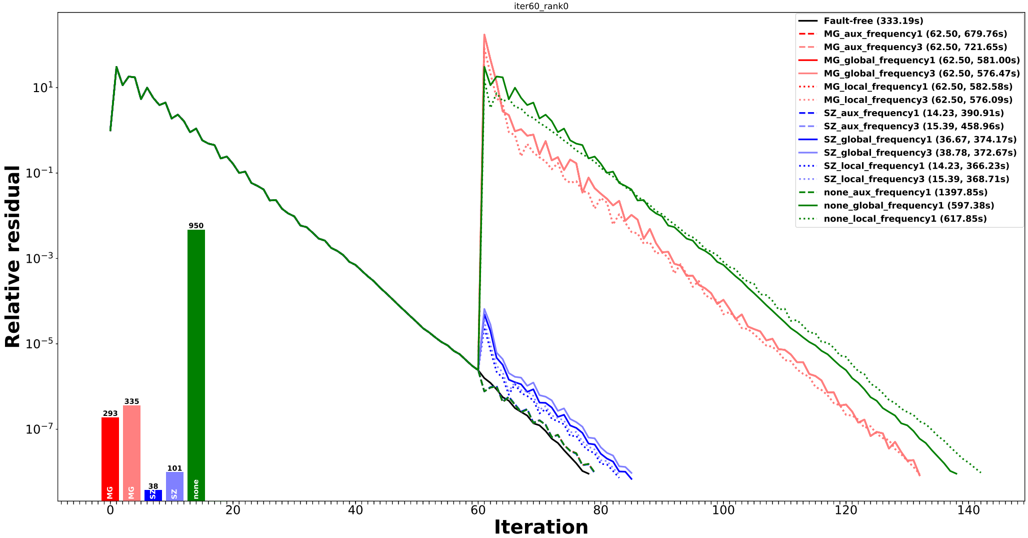


❶ Compressed checkpointing

Summary (early fault)

- Early on the advantage of using a backup is small ($\leq 10\%$ runtime)
- No backup: Global restart is better than local restoration with zeros
- With backup: Local restoration seems to be superior
- Lower backup frequency ($\rightarrow$ older backups) makes restoration worse
- MG compression is more robust with respect to age/delay
- Auxiliary problem can nearly restore convergence behaviour:
  - MG and SZ compressed initial solution reduces iteration count significantly
  - Age of backup is important
- In case of multiple data-losses a lower iteration count could be important

❶ Compressed checkpointing



❶ Compressed checkpointing

Summary (late fault)

- Multigrid compression seems not competitive in this scenario
- SZ compression works good and maintains a reasonable compression factor
- Local restoration works better than global roll-back
- A delay can deteriorate the quality of repair
  But: Local SZ restoration with frequency 3 is better than global roll-back with frequency 1 and has less communication and compression overhead
- Auxiliary solver improves quality significantly but increases runtime
- Greater delay/depth/compression still increases iteration count of auxiliary solver

Open task

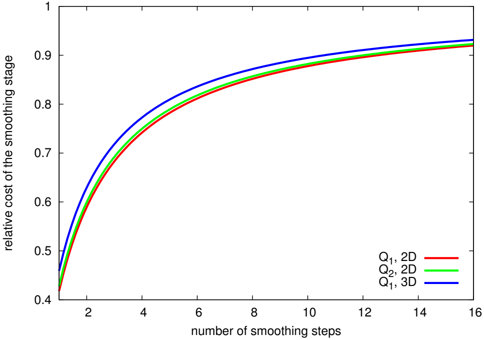
Develop a performance model to find the most effective combination

SDC-tolerant multigrid

Research goal

Increase the robustness of multigrid with respect to silent data corruption

❷ SDC-tolerant multigrid



Observation

Most time is spent within the smoothing stage

Idea

- Don't ensure correctness value by value
- Only verify if output of the smoothing stage is 'good enough'
- Use invariants of *Full Approximation Scheme multigrid* (FASMG) to test output
- Protect remaining part (transfer phase and coarse grid correction) with traditional checksums

❷ SDC-tolerant multigrid

STMG algorithm

Call : $\texttt{STMG}(k, \mathcal{A}, b, u^{(0)})$

$u^{(\nu)} = \mathcal{S}^{\nu}\left(u^{(0)}, b\right)$ // pre-smoothing

$r_k = b - \mathbf{A}_k u^{(\nu)}$

$\texttt{check\_and\_repair\_res}(r_k, k)$

$r_{k-1} = \mathbf{R}^{k}_{k-1} r_k$

$\tilde{v}^{(0)}_{k-1} = \mathbf{I}^{k}_{k-1} u^{(\nu)}$

$r_{k-1} = r_{k-1} + \mathbf{A}_{k-1} \tilde{v}^{(0)}_{k-1}$

$\tilde{v}_{k-1} = \texttt{STMG}(k-1, \mathcal{A}, r_{k-1}, \tilde{v}^{(0)}_{k-1})$ // coarse grid correction

$c = \tilde{v}_{k-1} - \tilde{v}^{(0)}_{k-1}$

$\texttt{check\_and\_repair\_cor}(c, k-1)$

$\tilde{u}^{(\nu)} = u^{(\nu)} + \mathbf{P}^{k-1}_{k}(c)$

$u = \mathcal{S}^{\mu}\left(\tilde{u}^{(\nu)}, b\right)$ // post-smoothing

if *on fine grid* **then**

    $\texttt{check\_and\_repair\_res}(b - \mathbf{A}_L u, k)$

end

- k denotes the current grid level
- $\mathbf{P}^{k-1}_{k}$ is the prolongation operator
- $\mathcal{S}^{\nu}$ is the smoother which is applied ν times
- Direct solver on coarsest grid

❷ SDC-tolerant multigrid

Check and repair algorithm (correction)

- Check output of smoother through element-wise comparison
- Threshold based on residual/correction norm (scaled by tolerance factor): Converges monotonously to zero if operator is s.p.d.
- Transfer (scale) to next level grid with transfer operator norm
- Store ‘faulty’ indices in set $\mathcal{L}$ and repair:

if $\mathcal{L} \neq \emptyset$ **then**
    $\tilde{v}_{k-1} = \mathbf{I}_{k-1}^{k} \tilde{v}_k^{(0)}$ `// restrict (non-faulty) initial approximation`
    $\tilde{c} = u_{k-1} - \tilde{v}_{k-1}$ `// calculate new correction vector`
    $\tilde{c} = \mathbf{P}_{k}^{k-1} \tilde{c}$ `// prolongate new correction vector`
    for $i \in \mathcal{L}$ **do**
        $c(i) = \tilde{c}(i)$ `// replace faulty components`
    end
end

- Residual check and repair works similar but easier
- Assumption: Coarse grid solver output is fault-free

❷ SDC-tolerant multigrid

Numerical tests

- V-cycle multigrid with $4+4$ Jacobi smoothing steps
- 1 million degrees of freedom, Q_1 Lagrange Finite Elements
- 4000 different fault scenarios per test problem
- Fault probability of 10^{-7} per degree of freedom
  $\Rightarrow$ Approximately once every 10th smoothing step on fine grid
  $\Rightarrow$ Approximately twice every multigrid iteration

	diff	**diff-conv**	**andiff**	**andiff-conv-reac**
fault-free	4	6	14	7
MG (div.)	4.225 (6.8%)	6.268 (8.4%)	15.111 (21.3%)	7.466 (11%)
STMG	4.038	6.007	14.007	7.017

- Nearly no false-positives: Approximately 15 in 4000 runs

M.A. and D. Göddeke, **Soft fault detection and correction for multigrid**, International Journal of High Performance Computing Applications, 2017

❷ SDC-tolerant multigrid

Numerical overhead

- Overhead of FASMG is approximately 20%
- Smoother protection itself results in an overhead of 4%
- Checksums lead to additional 5% ($8\times$ Jacobi smoothing)

	unprotected (MG)	unprotected (FASMG)	defect correction (checksums)	smoothing stage (new algorithm)	STMG (both)
time	35.49	43.02	45.23	44.76	46.18
factor	0.825	1	1.051	1.040	1.073
factor	1	1.212	1.274	1.261	1.301

$\Rightarrow$ Overall overhead of less than 30% compared to classical MG

❷ SDC-tolerant multigrid

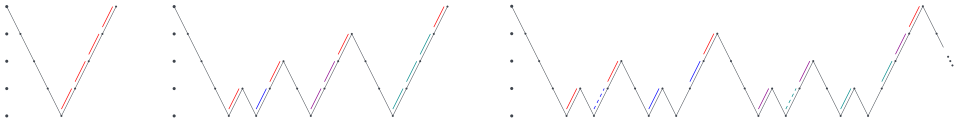
Applicability

- Geometric and algebraic multigrid (AMG)
- Standalone and as preconditioner
- Serial and parallel:

#it	17	18	19	20	21	25	34	41	div	avg
AMG	97	1			2	1	2	1	87	17.72
STAMG	179	4	6	2					0	17.12

Parallel execution of protected algorithm on 4 procs with CG and AMG preconditioner.

- All cycle types:



User level exception handling

Research goal

Extend the functionality of MPI for fault-tolerant algorithms

❸ User level exception handling

Challenges

- Detect locally thrown exceptions
- Inform all processes of the error
- Wrap it into a user-friendly C++ compliant interface
- Support asynchronous communication (similar to C++ future concept)

Code Example

```
try{ // scope to be protected
  Guard guard(communicator);
  Future f = init_communication();
  do_some_computation();
  f.get(); // MPI::Wait()
}catch(...) {
  // handle thrown exceptions
}
```

- Cheap guard object protects *try* block
- Is destructed during stack unwinding
- Propagate exception across communicator (uses `std::uncaught_exception`)

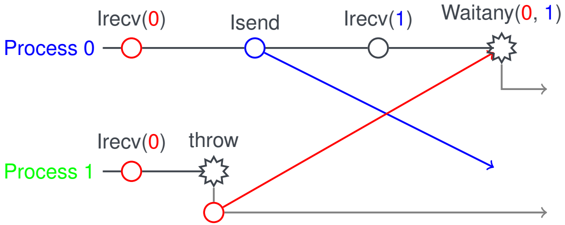
❸ User level exception handling

MPI-4 variant

- Interface is using the User-level failure-mitigation extension (ULFM)
- Provides functionality for
  - Hard fault *detection*
  - Communicator *revocation*
  - *Shrinking* of faulty communicator (i.e. excluding faulty processors)

MPI-3 variant

- Fall-back library which creates additional communication channel for exceptions
- Drawback: cannot interrupt MPI collectives, no hard fault protection



- `https://gitlab.dune-project.org/exadune/blackchannel-ulfm`

C. Engwer, M. A., N. Dreier, D. Göddeke, **A High-Level C++ Approach to Manage Local Errors, Asynchrony and Faults in an MPI Application**, Proceedings of PDP 2018, 2018

Summary and Outlook

Summary

- We developed three ‘orthogonal’ approaches to increase fault-tolerance, especially for multigrid-type algorithms:
  - Restoration with compressed checkpoints:
    MG and SZ compression both have their (dis-)advantages
  - Efficient SDC protection with built-in in properties
  - Exception-propagation to ensure same state in MPI programs
- ’User level exception handling’ can be used for many algorithms to develop MPI-4 ready fault-tolerant algorithms already in MPI-3
- Interface is ready for asynchronous algorithms (*future* concept):
  - Asynchronous checkpointing/repair
  - Local-failure local-recovery
  - . . .

What's next?

- Integrating the new MPI interface into DUNE[1]
- Improving features/functionality of the interface for wider applicability
- Local restoration for non-linear solvers
- Evaluating and combining developed concepts:
  - Switch between different compression and repair techniques: Adaptively select the most efficient one
  - Asynchronous checkpointing/repair
  - Asynchrony in multigrid with ideas from abstract Schwarz theory: Local smoothing while restoring lost processors?
  - …

Thinking about **ideas for fault-tolerance and asynchrony in remaining PDE solver parts**, not only linear solver

[1] funded by DFG: German Priority Programme 1648, SPPEXA, EXADUNE

Acknowledgements

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Multigrid and faults

Assumption: Multigrid is self-stabilising

Self-stabilising

Starting from any state the solver comes back to a valid state.

P. Sao, R. Vuduc, **Self-stabilizing Iterative Solvers**, 2013

- Original defined by Dijkstra in 1974 for systems of distributed control
- Examples: Newton- and Jacobi-methods

Multigrid and faults

Assumption: Multigrid is self-stabilising

Sketch of the proof

- Multigrid is a defect correction procedure, i.e., a fixed point iteration
- Hackbusch's multigrid convergence proof is based on contraction arguments:

If the contraction property holds for a given iteration operator, then convergence of the corresponding iteration procedure is guaranteed for any initial guess

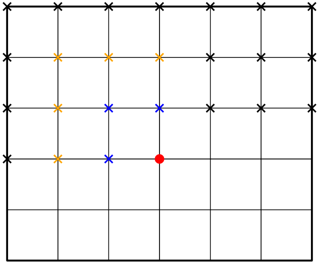
- Basically Banach's fixed point theorem
- The new initial guess is simply the last iterate with some faulty entries
- Matrices and grid transfer operators are fault-free
  $\Rightarrow$ Contraction property is not affected

❶ Compressed checkpointing

SZ compression (version 1.4.2, 2D)

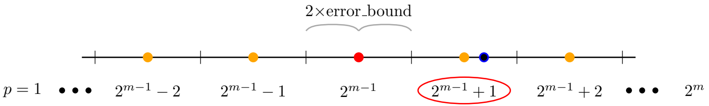
- Predict values row by row (top to bottom, left to right)
- $\mathcal{V} = \{V(i,j)\}$: set of already compressed point values
- Interpolation based first-phase prediction $f(i,j)$

1-Layer	$V(i,j-1)+V(i-1,j)-V(i-1,j-1)$
2-Layer	$2V(i,j-1)+2V(i-1,j)-4V(i-1,j-1)$ $-V(i,j-2)-V(i-2,j)+2V(i-2,j-1)$ $+2V(i-1,j-2)-V(i-2,j-2)$
...	...



× × × Processed points × × 2-Layer
× 1-Layer ● Next point

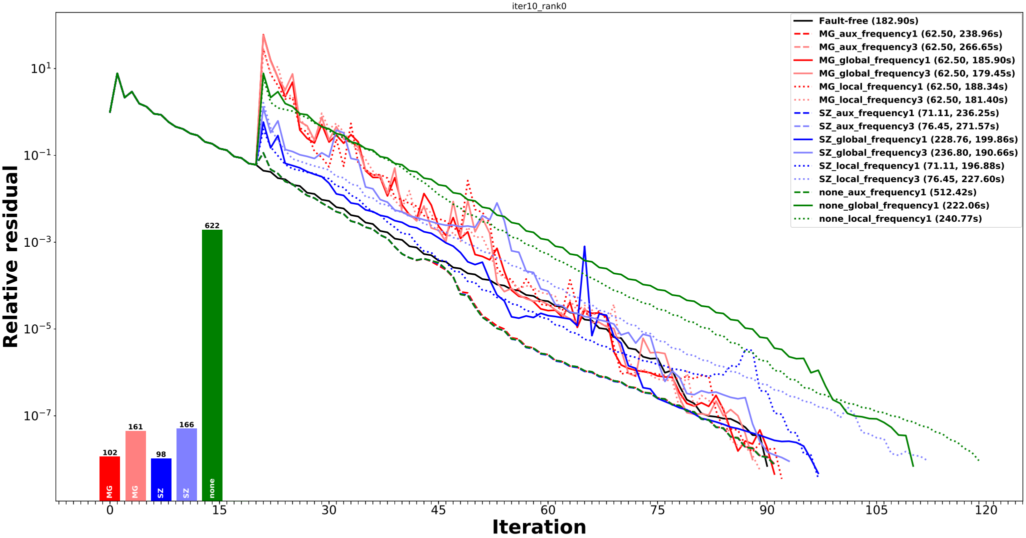
- 2^m intervals with size of $2\times$`error_bound` around $f(i,j)$



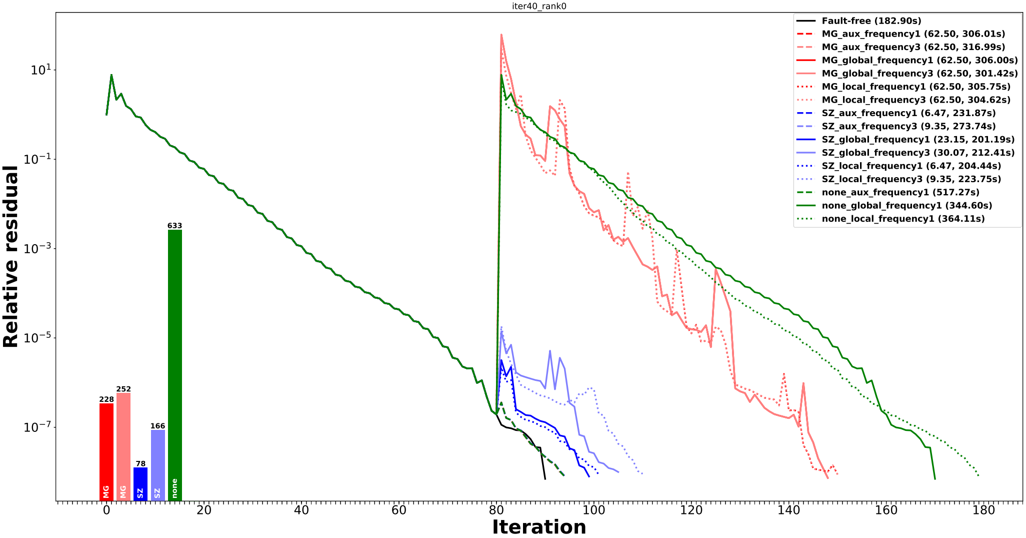
● first-phase prediction $f(i,j)$
● second-phase prediction
● real value

- Store index p or and $p=0$ and compressed binary-representation if the real value is not in any second-phase prediction interval
- Data is decompressed via interpolation and shifted by the Huffman-code

❶ Compressed checkpointing



❶ Compressed checkpointing



❷ SDC-tolerant multigrid

Differences between MG and FASMG

- Multigrid's correction problem is given by

$$\mathbf{A}_k(u_k + v_k) = b_k$$

- Classic MG uses linearity and searches on the next coarser level for the correction v_k only

$$\mathbf{A}_{k-1}v_{k-1} = \mathbf{R}^k_{k-1}(b_k - \mathbf{A}_k u_k)$$

- FASMG searches always for the full solution $\tilde{v}_k := u_k + v_k$

$$\mathbf{A}_{k-1}\tilde{v}_{k-1} = \mathbf{R}^k_{k-1}(b_k - \mathbf{A}_k u_k) + \mathbf{A}_{k-1}\mathbf{I}^k_{k-1}u_k$$

- $\tilde{v}_{k-1}$ is an approximation to the fine grid problem but with lower resolution
- We can interpret $\tilde{v}_{k-1}$ as a compressed backup of $\tilde{v}_k$
- $\mathbf{R}^k_{k-1}$ and $\mathbf{I}^k_{k-1}$ are different restriction operators

❷ SDC-tolerant multigrid

Justification of soft fault detection mechanism

- FAS correction problem

$$\mathbf{A}_{k-1}\tilde{v}_{k-1} = \mathbf{R}_{k-1}^{k} r_k + \mathbf{A}_{k-1}\tilde{v}_{k-1}^{(0)}$$

- Using linearity of the operator and $c_{k-1} = \tilde{v}_{k-1} - \tilde{v}_{k-1}^{(0)}$ yields

$$\|c_{k-1}\| \leq \|(\mathbf{A}_{k-1})^{-1}\|\|\mathbf{R}_{k-1}^{k}\|\|r_k\|$$

- $r_k = b_k - \mathbf{A}_k u_k^{(\nu)}$ on fine grid converges monotonously to zero if $\mathbf{A}$ is s.p.d.
- On coarser grid levels $b_k = \mathbf{R}_k^{k+1}(b_{k+1} - \mathbf{A}_{k+1}u_{k+1}^{(\nu)}) + \mathbf{A}_k\mathbf{I}_k^{k+1}u_{k+1}^{(\nu)}$ gives

$$\begin{aligned} r_k &= \mathbf{R}_k^{k+1}(b_{k+1} - \mathbf{A}_{k+1}u_{k+1}^{(\nu)}) + \mathbf{A}_k\mathbf{I}_k^{k+1}u_{k+1}^{(\nu)} - \mathbf{A}_k u_k^{(\nu)} \\ &= \mathbf{R}_k^{k+1}(b_{k+1} - \mathbf{A}_{k+1}u_{k+1}^{(\nu)}) + \mathbf{A}_k(u_k^{(0)} - u_k^{(\nu)}) \\ \Rightarrow \qquad \|r_k\| &\leq \|\mathbf{R}_k^{k+1}\|\|b_{k+1} - \mathbf{A}_{k+1}u_{k+1}^{(\nu)}\| + \|\mathbf{A}_k\|\,\|u_k^{(0)} - u_k^{(\nu)}\| \end{aligned}$$

- Operators are bounded, multigrid converges, smoothing property holds: $\|r_k\| \to 0$ and by this $\|c_k\| \to 0$